1.4 Universality

Yaneer Bar-Yam
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When we observe the largest scale behaviors of a system, we simplify the mathematical description of the system because there are fewer distinguishable states, and only a limited set of possible behaviors. This also means that systems that look different on a microscopic scale may not look different at the macroscopic scale, and their mathematical descriptions become the same.

An important example of this arose in the study of phase transitions using the new mathematics of renormalization group. The transition when boiling a liquid to a gas has the same properties as the one that occurs when a heating a magnet up to the point where it becomes non-magnetic (ferromagnet to paramagnetic transition). Magnets have local magnetizations that fluctuate and interact at a critical point just like local changes of density at the water to vapor critical point. The result is that these two seemingly different types of systems map mathematically onto each other.

As renormalization group was more widely applied, other instances were found of systems that have the same behavior even though they differ in detail, a concept that became referred to as *universality*. Still, while many systems have the same behavior, there are multiple distinct behaviors. Together this means that systems fall into classes of behaviors, leading to the term '*universality class*.' Since renormalization group focuses on how behaviors transform across scales leading to power laws, the value of the power law exponent became used as a signature of the universality class.

In a sense, the idea that many systems can be described by the same large scale behavior is used in traditional theory. Scientists use the normal distribution for many different biological and social systems. Any system having sufficiently independent components, satisfies the axioms of the central limit theorem, and therefore can be described by the normal distribution. When there are dependencies, the normal distribution no longer applies, but there are other behaviors that are characteristic of other kinds of dependencies. To study those behaviors, we have to determine the way different kinds of dependencies give rise to kinds of large scale behavior.

There are even more basic ways a common mathematical description of systems is used, e.g., point particle motion describes the motion of many distinct objects, and wave equations describe everything from music strings to water waves to light. Even though the specific systems are very different, the dependencies that give rise to their behaviors, and the behaviors themselves, are related mathematically.

How does universality work for complex systems? Unlike traditional renormalization group, we do not consider the limit of infinite size and power law exponents. Instead, the states of our representation must correspond to the states of the system at the scale of observation. Moreover, instead of describing the equilibrium energy, we describe dynamics and system response. The mathematical representation of one system at a particular scale may correspond to the behavior of other systems despite different underlying components. This is a general concept of universality (Fig. 1).

What are the cases where the thermodynamic limit does not serve to expose universality? An important example is pattern formation that results in spots and stripes, like those on predator and prey animals. This type of pattern formation was described by Alan Turing and are called Turing patterns (Fig. 1). They arise in many ways, for example from the reaction of diffusing chemical species. If we think about what happens with a very large pattern we see that at large enough scales, these patterns look only gray. Still, we can map these descriptions from system to system. The patterns represent universal classes of behavior. Microscopic changes only change the pattern to the extent that they change the relevant parameters of those patterns.

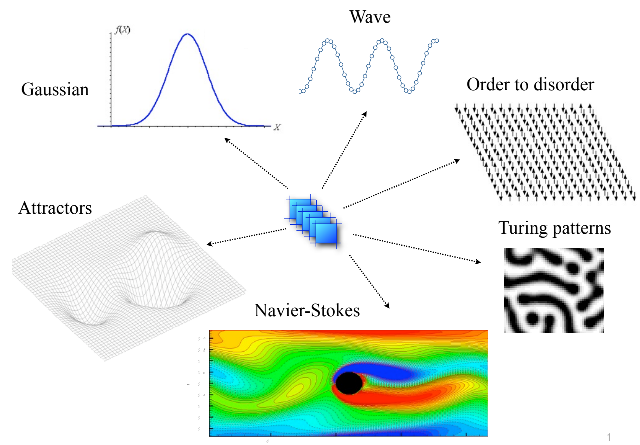

FIG. 1: When we focus on the largest scale, system behaviors map onto simplified models, each of which applies to a large set of possible systems with widely different microscopic details. Examples shown in this figure: the Gaussian distribution, wave motion, order to disorder transitions, Turing patterns, fluid flow described by Navier-Stokes equations, attractor dynamics. That only a few models capture the behavior of a wide range of systems underlies the idea of universality—systems are members of universality classes of behavior.

The adoption of Turing's ideas in biology for patterns on animal skins has been controversial precisely because the pattern dynamics does not capture microscopic mechanisms. This controversy misses the key point about universality. Universality should be intuitive as we don't need to describe the molecular processes to characterize the variation between patterns on species, or individual members of a species, or the dynamics of a pattern as it forms, and do not affect roles of these patterns in social and ecological interactions. This is similar to the ability to describe planetary motion without describing details of individual planet structure.

The study of universality enables us to identify classes of systems whose behaviors can be described the same way by a common mathematical model. This is the principle of universality that is formalized by renormalization group and generalized by multiscale information theory to the scientific study of complex systems.